§ 01 QUADRATIC EQUATIONS & PARABOLA DYNAMICS

CORE AXIOM

$$\text{Standard Form: } ax^2 + bx + c = 0 \implies \text{Quadratic Formula: } x = \left[-b \pm \sqrt{(b^2 - 4ac)} \right] / (2a)$$

DISCRIMINANT ($\Delta = B^2 - 4AC$)

- **$\Delta > 0$:** 2 Distinct Real Roots
- **$\Delta = 0$:** 1 Repeated Real Root (Tangent)
- **$\Delta < 0$:** 2 Complex Roots ($a \pm bi$)

PARABOLA VERTEX & AXIS

- **Axis of Symmetry:** $x = -b / (2a)$
- **Vertex (h, k):** $h = -b/(2a)$, $k = c - b^2/(4a)$
- **Vertex Form:** $y = a(x - h)^2 + k$

VIETA'S ROOT FORMULAS

For roots r_1, r_2 of $ax^2 + bx + c = 0$:

- **Sum of Roots:** $r_1 + r_2 = -b / a$
- **Product of Roots:** $r_1 \cdot r_2 = c / a$

§ 02 FACTORING & POLYNOMIAL EXPANSION

SHORTCUTS

DIFF OF SQUARES	$a^2 - b^2 = (a - b)(a + b)$
SQUARE TRINOMIAL	$(a \pm b)^2 = a^2 \pm 2ab + b^2$
SUM OF CUBES	$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
DIFF OF CUBES	$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
BINOMIAL CUBE	$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

§ 03 REMAINDER & FACTOR THEOREMS

EXAM HACK

$$\text{Division Algorithm: } P(x) = (x - c) \cdot Q(x) + R \implies P(c) = R$$

- **Remainder Theorem:** Dividing $P(x)$ by $(x - c)$ gives remainder = **$P(c)$**.
- **Factor Theorem:** $(x - c)$ is a factor of $P(x)$ **if and only if $P(c) = 0$**.
- **Rational Root:** p/q has p | constant a_0, and q | leading coefficient a_n.

SIGN ALERT	Divisor $(x + 4) \implies$ Evaluate $c = -4$!

§ 04 EXPONENTS & LOGARITHM LAWS

LAWS

PRODUCT RULE	$\log_b(xy) = \log_b(x) + \log_b(y)$
QUOTIENT RULE	$\log_b(x/y) = \log_b(x) - \log_b(y)$
POWER RULE	$\log_b(x^k) = k \cdot \log_b(x)$
CHANGE OF BASE	$\log_b(x) = \ln(x) / \ln(b)$
CONTINUOUS GROWTH	$A = P \cdot e^{rt}$ | $A = P(1 + r/n)^{nt}$

§ 05 SEQUENCES, SERIES & COMBINATORICS

SERIES

ARITHMETIC TERM	$a_n = a_1 + (n - 1)d$
ARITH SUM S_N	$S_n = (n/2)[\, 2a_1 + (n-1)d \,] = n(a_1+a_n)/2$
GEOMETRIC TERM	$a_n = a_1 \cdot r^{n - 1}$
INFINITE GEOM SUM	$S_\infty = a_1 / (1 - r)$ [for $\lvert r \rvert < 1$]
BINOMIAL TERM	$T_{k+1} = \binom{n}{k} \cdot a^{n-k} \cdot b^k$

§ 06 UNIT CIRCLE HIGH-YIELD ANGLE VALUES (X = COS Θ, Y = SIN Θ)

MUST MEMORIZE

0° (0 rad)	30° (π/6)	45° (π/4)	60° (π/3)	90° (π/2)
cos: 1	cos: √3/2	cos: √2/2	cos: 1/2	cos: 0
sin: 0	sin: 1/2	sin: √2/2	sin: √3/2	sin: 1
tan: 0	tan: √3/3	tan: 1	tan: √3	tan: undefined

ASTC QUADRANT SIGNS: Q1: All Pos | Q2: Sin Pos | Q3: Tan Pos | Q4: Cos Pos

RADIANS ↔ DEGREES: rad = deg · (π/180) | deg = rad · (180/π)

§ 07 PYTHAGOREAN & QUOTIENT IDENTITIES

FUNDAMENTAL

PRIMARY PYTH	$\sin^2\theta + \cos^2\theta = 1$
TANGENT PYTH	$1 + \tan^2\theta = \sec^2\theta$
COTANGENT PYTH	$1 + \cot^2\theta = \csc^2\theta$
RECIPROCALS	$\sec\theta = 1/\cos\theta$ | $\csc\theta = 1/\sin\theta$
EVEN / ODD	$\sin(-\theta) = -\sin\theta$ | $\cos(-\theta) = \cos\theta$

§ 08 DOUBLE, HALF & COMPOUND ANGLES

FORMULAS

DOUBLE SIN	$\sin(2\theta) = 2\sin\theta\cos\theta$
DOUBLE COS	$\cos(2\theta) = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$
DOUBLE TAN	$\tan(2\theta) = 2\tan\theta / (1 - \tan^2\theta)$
SIN ADDITION	$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$
COS ADDITION	$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

§ 09 NON-RIGHT TRIANGLE LAWS

OBLIQUE

LAW OF SINES	$a / \sin(A) = b / \sin(B) = c / \sin(C)$
LAW OF COSINES	$c^2 = a^2 + b^2 - 2ab \cdot \cos(C)$
TRIANGLE AREA	$\text{Area} = (1/2)\, a \cdot b \cdot \sin(C)$
HERON FORMULA	$\text{Area} = \sqrt{[\, s(s-a)(s-b)(s-c)\,]}$ $(s = \text{perimeter}/2)$

§ 10 CIRCLE THEOREMS & COORDINATE GEOMETRY

GEOMETRY

CIRCLE EQ	$(x - h)^2 + (y - k)^2 = r^2$ [Center (h, k)]
ARC & SECTOR	$s = r\theta$ | $\text{Area} = (1/2)\, r^2\, \theta$ [θ in radians]
INSCRIBED ANGLE	Inscribed Angle = (1/2) Intercepted Arc
TANGENT–SECANT	$(\text{Tangent Segment})^2 = \text{External} \cdot \text{Whole Secant}$

§ 11 DIFFERENTIAL CALCULUS SHORTCUT RULES & COMMON DERIVATIVES

DERIVATIVES

Limit Definition: f'(x) = lim (h → 0) [f(x + h) - f(x)] / h ⟹ Instantaneous Rate of Change

POWER RULE	d/dx [x^n] = n · x^(n - 1)
PRODUCT RULE	d/dx [u · v] = u'v + uv'
QUOTIENT RULE	d/dx [u / v] = [u'v - uv'] / v²
CHAIN RULE	d/dx [f(g(x))] = f'(g(x)) · g'(x)
EXP LAW	d/dx [e^(kx)] = k · e^(kx) | a^x ln a
LN / LOG	d/dx [ln x] = 1/x | 1/(x ln a)
SIN & COS	d/dx [sin x] = cos x | [cos x] = -sin x
TAN & SEC	d/dx [tan x] = sec²x | [sec x] = sec x tan x

§ 12 INTEGRAL CALCULUS & FUNDAMENTAL THEOREM

INTEGRALS

FTC RULE	∫_a^b f(x) dx = F(b) - F(a) [F'(x) = f(x)]
POWER INTEGRAL	∫ x^n dx = [x^(n+1) / (n+1)] + C (n ≠ -1)
RECIPROCAL	∫ (1/x) dx = ln |x| + C
INT BY PARTS	∫ u dv = u · v - ∫ v du
EXP INTEGRAL	∫ e^(kx) dx = (1/k) e^(kx) + C
TRIG INTEGRAL	∫ sin x dx = -cos x + C | ∫ cos x dx = sin x + C

§ 13 PROBABILITY & STATISTICS ESSENTIALS

STATISTICS

VARIANCE S²	s² = Σ(x_i - x)² / (n - 1)
STD DEV Σ	σ = √[Σ(x_i - μ)² / N]
Z-SCORE	z = (x - μ) / σ
BELL CURVE	±1σ: 68.2% | ±2σ: 95.4% | ±3σ: 99.7%
BAYES RULE	P(A|B) = [P(B|A) · P(A)] / P(B)

§ 14 MATRIX ALGEBRA & DISCRETE MATH

MATRICES

DET(2×2)	det([a,b;c,d]) = ad - bc
INV A⁻¹	A^(-1) = (1/det) · [d, -b; -c, a]
PERM NPR	nPr = n! / (n - r)!
COMB NCR	nCr = n! / [r!(n - r)!]
EULER FORM	e^(iθ) = cos θ + i sin θ ⟹ e^(iπ) + 1 = 0